Proper elements for resonant planet-crossing orbits

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Introduction

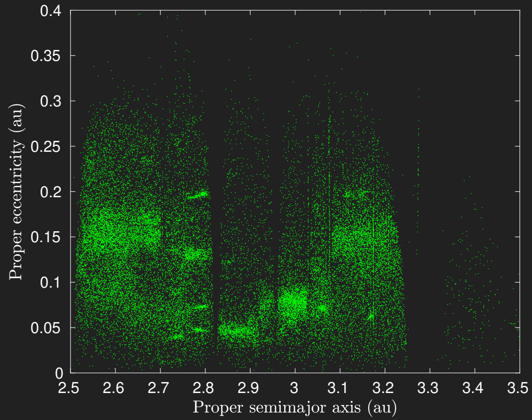
Proper elements are **quasi-integrals** of motion of the N-body problem

Methods of computation:

- Analytical methods (e.g. *Hirayama 1918, Kozai 1962, Milani & Knežević 1990, 1992, 1994*)
- Semi-analytical methods (e.g. *Williams 1969, 1979, Lemaitre & Morbidelli 1994*)
- Synthetic methods (*Knežević & Milani 2000, 2003*)

Applications:

- Identify asteroid families
- Compute family ages

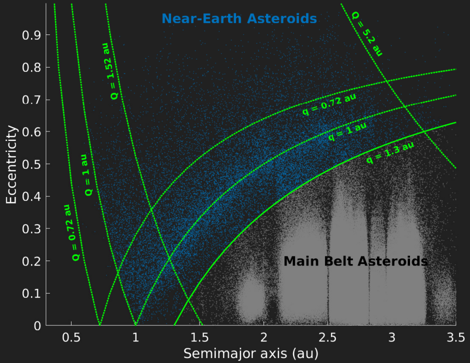


NEOs proper elements

Near-Earth objects (NEOs) can **cross** the orbits of planets

Problems:

- Orbit crossings cause divergence of series
- Orbit crossings cause divergence of quadratures
- Close encounters with planets shorten the Lyapunov time



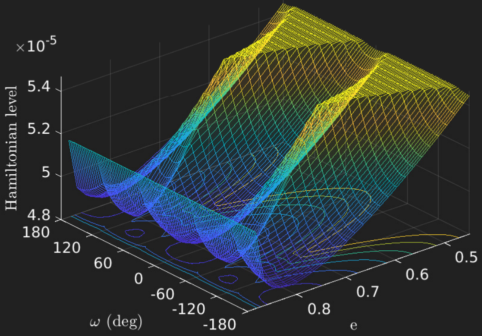
NEOs proper elements

Gronchi & Milani 2001:

- Kozai secular model

$$\mathcal{H} = \frac{\kappa^2}{(2\pi)^2} \sum_{i=1}^{N} \int_0^{2\pi} \int_0^{2\pi} \frac{\mu_i}{|\mathbf{r} - \mathbf{r}_i|} \mathrm{d}\ell \mathrm{d}\ell_i$$

- **No MMRs** - No close encounters
- Continuation of solutions beyond crossing



Proper elements:

- $e_{\min}, e_{\max}, i_{\min}, i_{\max}$
- frequencies $s, g - s$ of Ω, ω

Catalog: NEODyS

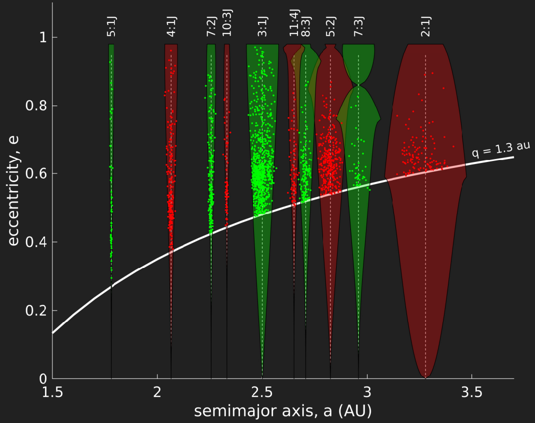
Motivations and goals

Motivations:

- Many NEOs are in a mean-motion resonance (MMR) today
- MMR could significantly affect the evolution
- No theory of proper elements for resonant NEOs available

Goals:

- Define proper elements for resonant NEOs
- Understand the importance of MMRs in the long-term dynamics of NEOs



Semi-secular model

Model:

- Circular restricted N-body problem
- κ Gauss constant
- μ_i masses of the planets
- $\varepsilon = \mu_5$ small parameter

Delaunay variables:

$$\begin{cases} L = \kappa\sqrt{a}, \\ G = L\sqrt{1-e^2}, \\ Z = G\cos i, \end{cases} \qquad \begin{cases} \ell = M, \\ g = \omega, \\ z = \Omega. \end{cases}$$

Hamiltonian:

$$\widetilde{\mathcal{H}} = \underbrace{\boxed{-\frac{\kappa^4}{2L^2}}}_{\mathcal{H}_0} + \underbrace{\boxed{\sum_{j=1}^{8} \mathrm{n}_j L_j}}_{\text{Extended space}} + \varepsilon \underbrace{\boxed{\left[-\kappa^2 \sum_{j=1}^{8} \frac{\mu_j}{\mu_5} \left(\frac{1}{|\mathbf{r} - \mathbf{r}_j|} - \frac{\mathbf{r}\cdot\mathbf{r}_j}{|\mathbf{r}_j|^3} \right) \right]}}_{\mathcal{H}_1}$$

Semi-secular model

Assumption: $h{:}h_p$ MMR with the p-th planet, critical argument

$$\sigma = h\underbrace{\boxed{\lambda}}_{\ell+\omega+\Omega} - h_p\underbrace{\boxed{\lambda_p}}_{\ell_p+\omega_p+\Omega_p} - (h-h_p)\underbrace{\boxed{\varpi}}_{\omega+\Omega}$$

Resonant variables (from *Saillenfest et al. 2016*):

$$\begin{cases} \Sigma = \frac{L}{h} \\ \Gamma = hL - h_p L_p \\ U = G - \frac{h}{h_p}L \\ V = Z - \frac{h}{h_p}L \end{cases} \qquad \begin{cases} \sigma = h\lambda - h_p\lambda_p - (h-h_p)\varpi \\ \gamma = c\ell + c_p(\varpi - \ell_p) \\ u = \omega \\ v = \Omega \end{cases}$$

Semi-secular Hamiltonian:

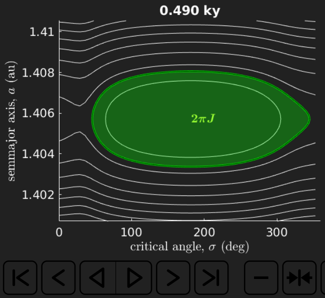
$$\mathcal{K} = \underbrace{\boxed{-\frac{\kappa^4}{2(h\Sigma)^2} - \mathrm{n}_p h_p \Sigma}}_{\mathcal{K}_0} + \varepsilon\Bigg[\underbrace{\boxed{\frac{\kappa^2}{\mu_5}\sum_{\substack{j=1\\ j\neq p}}^{N}\frac{\mu_j}{(2\pi)^2}\int_0^{2\pi}\int_0^{2\pi}\frac{-1}{|\mathbf{r}-\mathbf{r}_j|}\mathrm{d}\ell\,\mathrm{d}\ell_j}}_{\mathcal{K}_{\text{sec}}} + \underbrace{\boxed{\frac{\kappa^2}{\mu_5}\frac{\mu_p}{2\pi}\int_0^{2\pi}\left(\frac{\mathbf{r}\cdot\mathbf{r}_p}{|\mathbf{r}_p|^3} - \frac{1}{|\mathbf{r}-\mathbf{r}_p|}\right)\mathrm{d}\gamma}}_{\mathcal{K}_{\text{res}}}\Bigg]$$

Remark: crossing singularity is treated as in *Gronchi & Tardioli 2013*

Secular model

Remark: $\nu_\sigma \propto \sqrt{\varepsilon}$ and $\nu_u \propto \varepsilon$

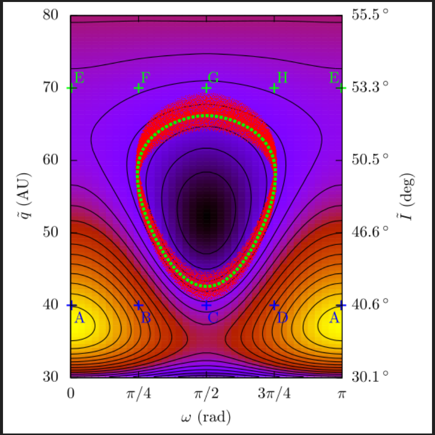
If $\nu_u/\nu_\sigma \ll 1 \Rightarrow$ adiabatic theory $(\Sigma, \sigma) \mapsto (J, \theta)$



Secular Hamiltonian:

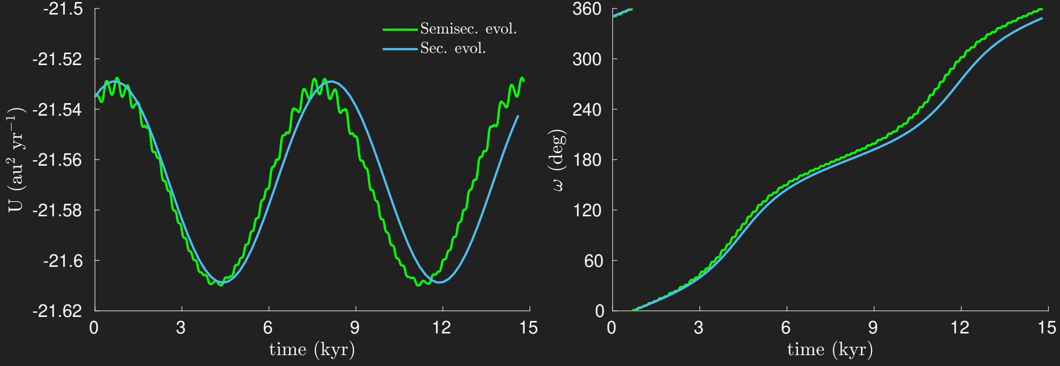
$$\mathcal{F}(J, U, V, u) = \mathcal{K}(\Sigma_0, U, V, \sigma_0, u)$$

Level curves of $\mathcal{F}$
(from *Saillenfest et al. 2016*)



Secular model

Example: (887) Alinda - 3:1J MMR - Mars crosser



Proper elements computation

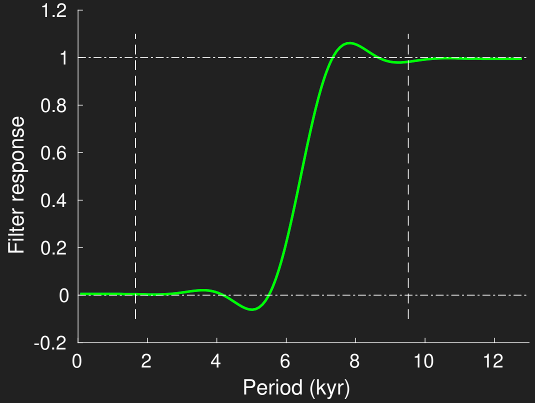
(1) Compute initial elements for the semi-secular Hamiltonian
- Propagate the osculating orbit for short time
- Filter out short periodic oscillations ($P \sim 200$ yr)

(2) Propagate the semi-secular dynamics for 200 ky

(3) Filter out short periodic oscillations ($P \sim 10$ ky)

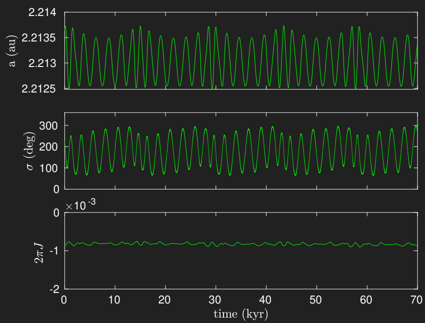
(4) Determine proper frequencies $s, g - s$ of Ω, ω

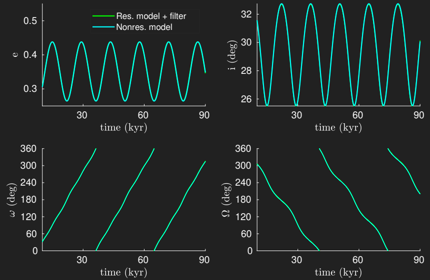
(5) Compute $e_{\text{max}}, e_{\text{min}}, i_{\max}, i_{\min}$



Example 1

Object: (159560) 2001 TO103 - 4:7M MMR - $2\pi J$ constant - same res. and non-res. dynamics

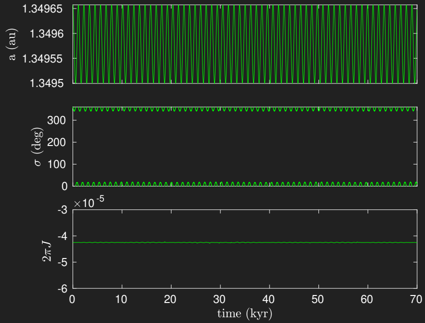


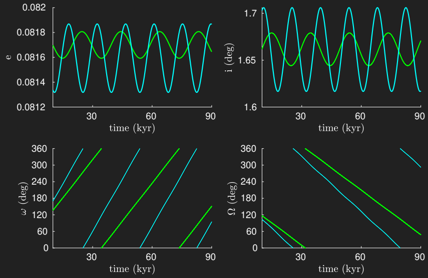


Model	$e_{\min}$	$e_{\max}$	$i_{\min}$ (°)	$i_{\max}$ (°)	$g - s$ (″/yr)	s (″/yr)
Res	0.2650	0.4380	25.50	32.70	45.411	-36.322
Non-Res	0.2649	0.4385	25.522	32.749	45.419	-36.367

Example 2

Object: (138911) 2001 AE2 - 5:6M MMR - $2\pi J$ constant - different res. and non-res. dynamics

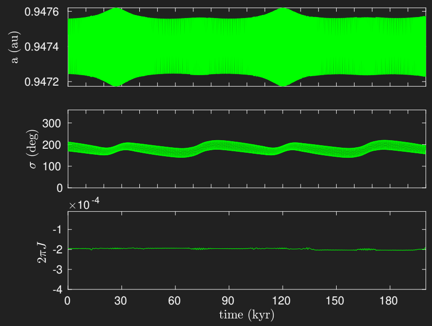


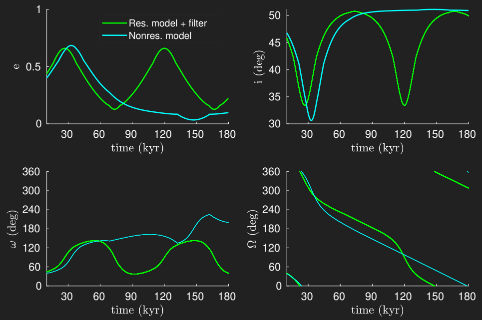


Model	$e_{\min}$	$e_{\max}$	$i_{\min}$ (°)	$i_{\max}$ (°)	$g-s$ (″/yr)	s (″/yr)
Res	0.0816	0.0818	1.64	1.68	33.07	-19.30
Non-Res	0.0813	0.0819	1.616	1.706	45.227	-23.913

Example 3

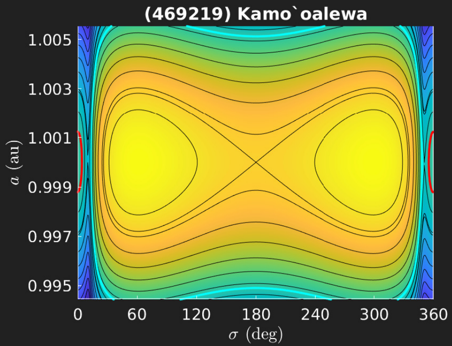
Object: (5381) Sekhmet - 2:3V MMR - V/E crosser - $2\pi J$ constant - different res. and non-res. dynamics

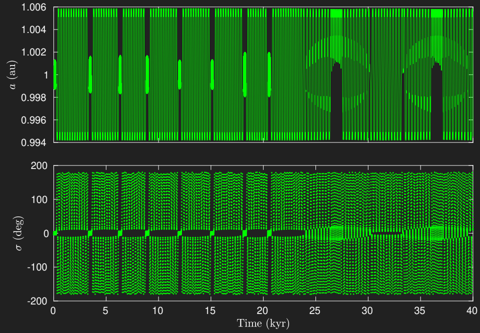




Example 4

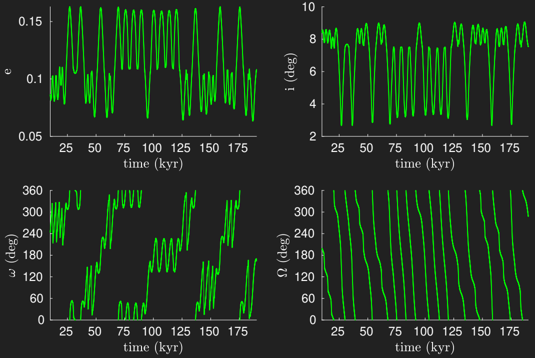
Object: (469219) Kamo'oalewa - 1:1E MMR - chaotic secular resonant dynamics

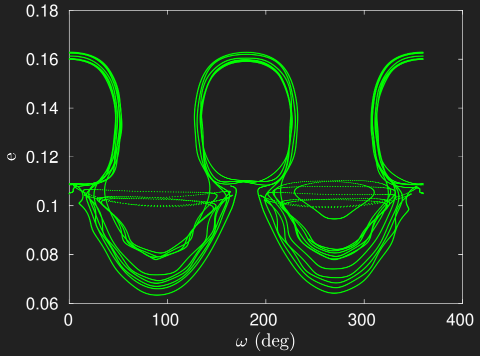




Example 4

Object: (469219) Kamo'oalewa - 1:1E MMR - chaotic secular resonant dynamics





Conclusions

- We defined and implemented an algorithm for the computation of proper elements of resonant NEOs
- MMRs affect the long-term evolution of NEOs

Future works

- Estimate the timespan of validity of proper elements
- Identify NEOs affected by MMRs
- Catalog of proper elements of resonant NEOs