Sekularne rezonance

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Sekularne rezonance: definicija

- Sekularne rezonance su singulariteti u faznom prostoru elemenata kretanja gde linearne kombinacije frekvencija longituda perihela i čvora poremećenog (asteroid) i poremećajnih (planete) tela teže nuli.

- g - frekvencija longitude perihela

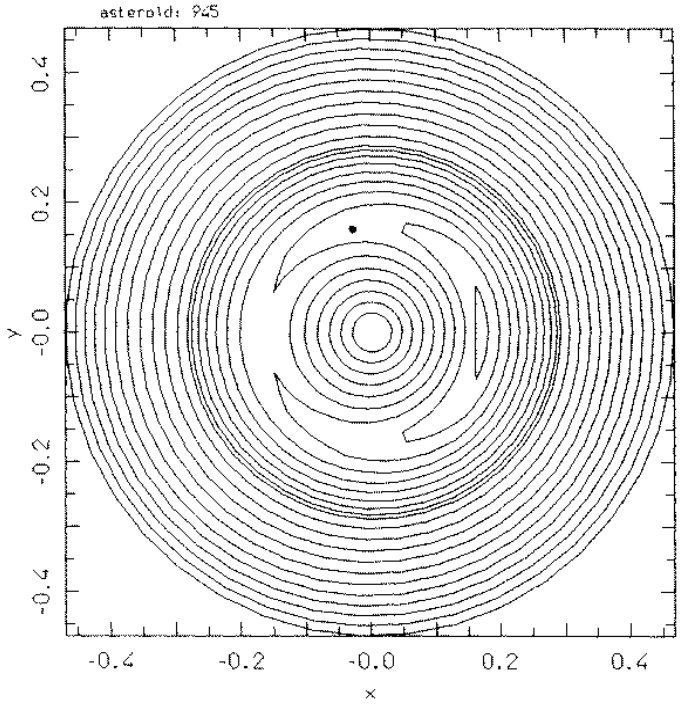
$$W^{(4)} = -\sum_{h} \frac{H^{(2)}_{4h}}{h \cdot \nu_0} \sin(h \cdot \beta)$$

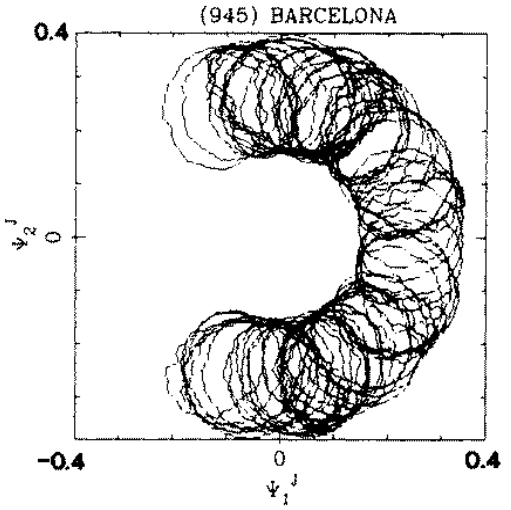
- s – frekvencija longitude čvora
- Dozvoljene kombinacije određene D'Alamberovim pravilima.

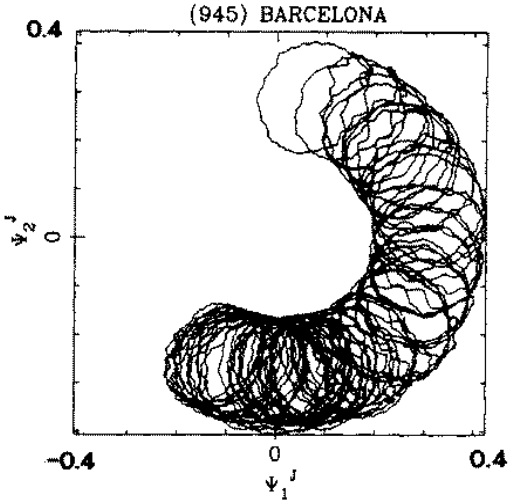
- Linearne rezonance (Jup + Sat): $g - g_5$; $g - g_6$; $s - s_6$
- Nelinearne rezonance: $g + s - g_6 - s_6$; $2g - g_5 - g_6$; $g+2s-g_5-2s_6$

- (g_5 = 4.26 "/y ; g_6 = 28.25 "/y; s_6 = -26.34 "/y)

Portret rezonantnog faznog prostora u sekularnoj rezonanci g – g_5







No.	Divisor
1	$g - g_5$
2	$g - g_6$
3	$s - s_6$
4	$g - g_5 + s - s_6$
5	$g - g_6 + s - s_6$
6	$2g - 2s$
7	$g - 2g_5 + g_6$
8	$g + g_5 - 2g_6$
9	$2g - g_5 - g_6$
10	$g - g_5 - s + s_6$
11	$g - g_6 - s + s_6$
12	$2g - s - s_6$
13	$g + g_5 - 2s$
14	$g + g_6 - 2s$
15	$2g - 2s_6$
16	$g + g_5 - s - s_6$
17	$g + g_6 - s - s_6$
18	$g + g_5 - 2s_6$
19	$g + g_6 - 2s_6$
20	$s - s_6 + g_5 - g_6$
21	$s - s_6 - g_5 + g_6$
22	$2s - 2g_5$
23	$2s - 2g_6$
24	$2s - g_5 - g_6$
25	$s + s_6 - 2g_5$
26	$s + s_6 - 2g_6$
27	$s + s_6 - g_5 - g_6$
28	$g_5 - g_6$

SECULAR DIVISORS APPEARING UP TO DEGREE 4 IN THE INVARIABLE PLANE FRAME

Milani A. and Knežević, Z. 1990, Secular perturbation theory and computation of asteroid proper elements, CMDA , 49, 347-411

Sekularne rezonance do 6 stepena

20 rezonanci kao kombinacije linearnih

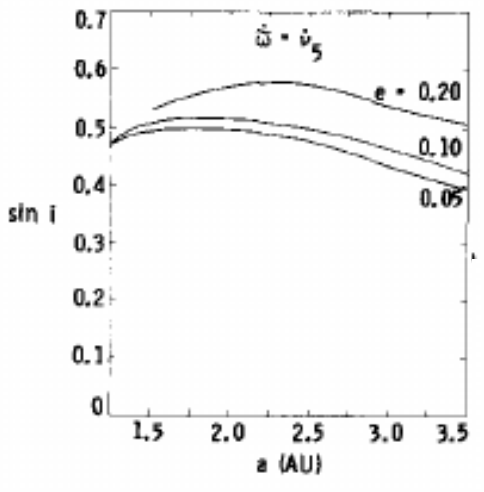
3g-2g5-g6
3g-g5-2g6
2g+s-g5-g6-s6
2g-s-g5-g6+s6
2g+s-2g5-s6
2g-s-2g5+s6
2g+s-2g6-s6
2g-s-2g6+s6
g+2s-g5-2s6
g-2s-g5+2s6
g+2s-g6-2s6
g-2s-g6+2s6
2g-3g5+g6
2g+g5-3g6
g+s-2g5+g6-s6
g+s+g5-2g6-s6
g-s-2g5+g6+s6
g-s+g5-2g6+s6
2s-g5+g6-2s6
2s+g5-g6-2s6

(z2)

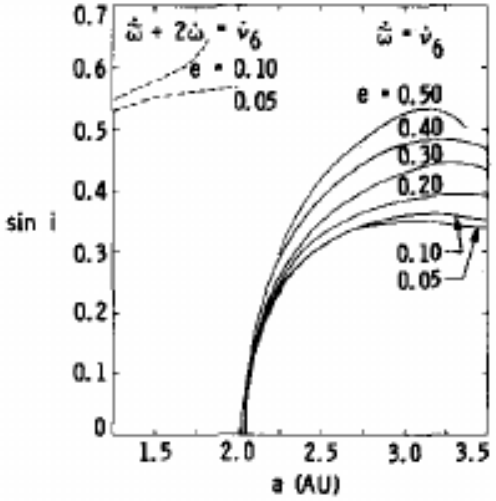
13 dodatnih rezonanci kao kombinacije sa Kozai rezonancom 2g – 2s

3g-2s-g5
3g-2s-g6
3s-2g-s6
3s-g-g5-s6
3g-s-g5-s6
3s-g-g6-s6
3g-s-g6-s6
2g-2s-g5+g6
2g-2s+g5-g6
2s-g-2g5+g6
2s-g+g5-2g6
g+2s-g5-2g6
g+2s-2g5-g6

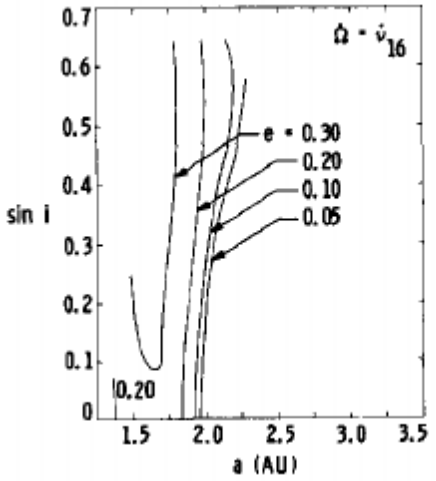
ν_5 ($g = g_5$)



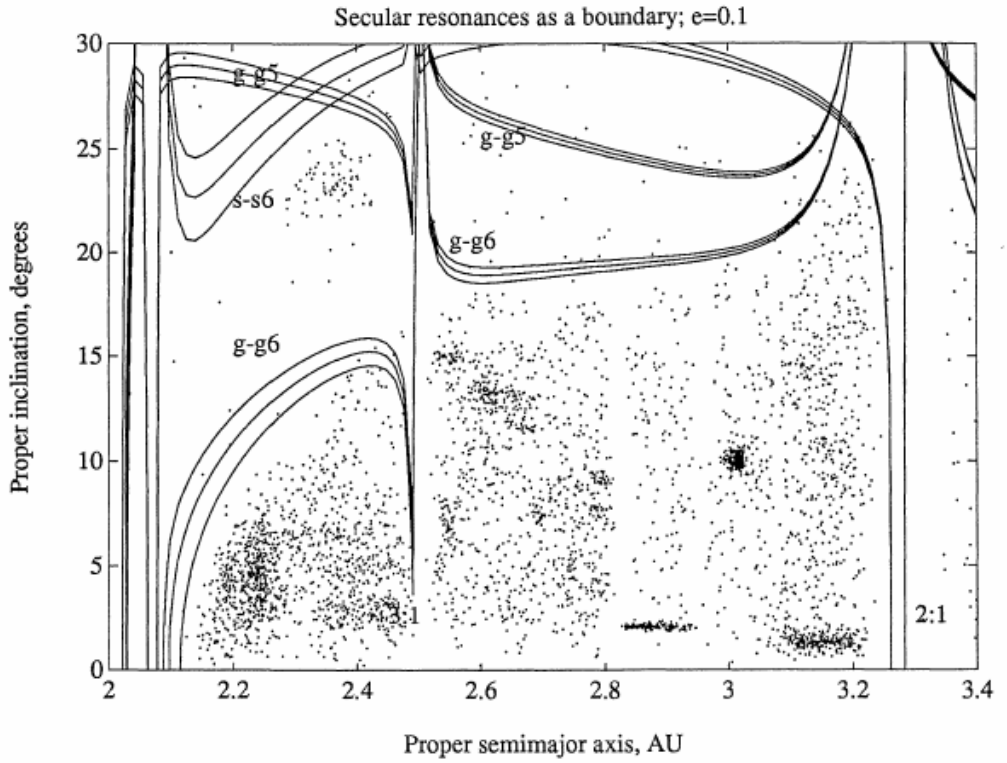
ν_6 ($g = g_6$)

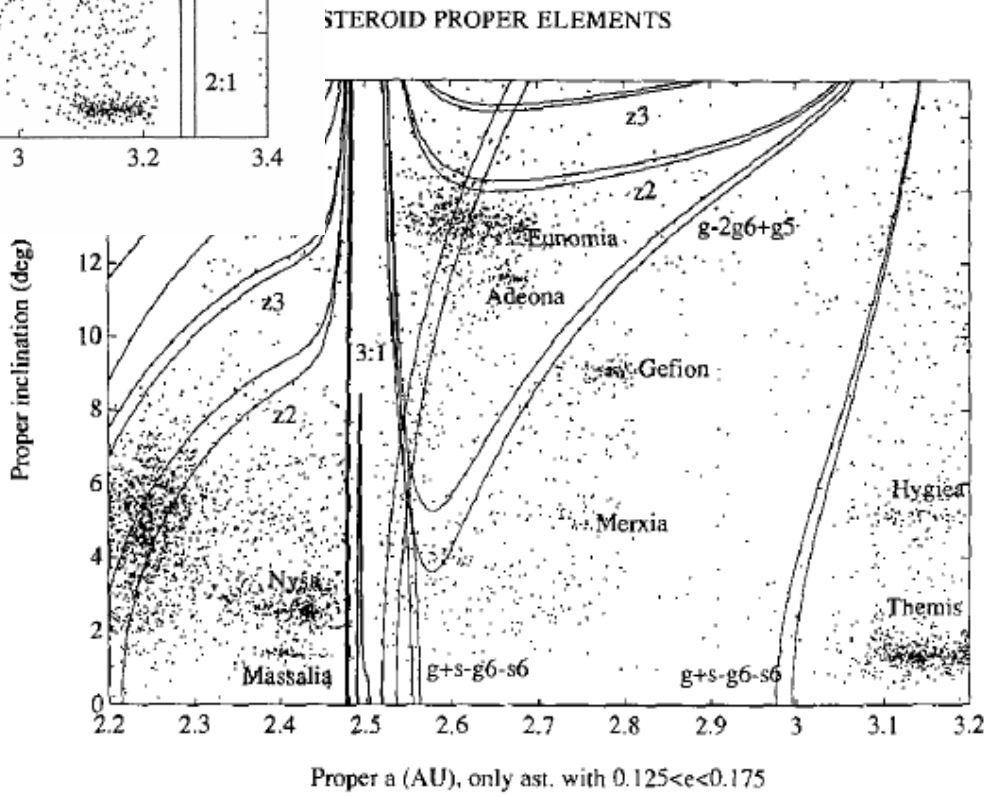


ν_{16} ($s = s_6$)



Williams, J.G., Faulkner, J.: 1981,
The positions of secular resonance surfaces.
Icarus 46, 390-399.





Milani & Knežević, 1990

Položaji sekularnih rezonanci

Poznavanje preciznih položaja sekularnih rezonanci važno je zbog:

- interakcije sa asteroidima unutar ili u blizini rezonantnih zona,
- preoblikovanja asteroidnih familija u interakciji sa rezonancama,
- transport objekata unutar ili van granica asteroidnog pojasa, ….

Direktne metode:
Williams, 1969, Yoshikawa, 1987
Morbidelli and Henrard, 1991a,b , Michel and Froeschle, 1997

Milani and Knežević, 1990,1992, 1994, Knežević et al. 1991

Indirektne metode:
Michtchenko et al 2010, Carruba 2010
Carruba 2007, Novaković and Tsirvoulis 2016, Milani et al. 2017

Zadatak

Poboljšati tačnost položaja sekularnih rezonanci pomoću jednostavne direktne metode “interpolacije na koordinatnoj mreži”, uz korišćenje najbolje raspoložive, tzv. sintetičke, teorije za računanje sekularnih frekvencija, za šta je potrebno naći način da se svakom čvoru mreže pridruže odgovarajuće vrednosti frekvencija.

Sekularne frekvencije su u analitičkoj teoriji date kao eksplicitne funkcije sopstvenih elemenata, pa se svaka njihova konkretna vrednost direktno preslikava u jednoznačno definisane konkretne vrednosti frekvencija.*

U sintetičkoj teoriji one se dobijaju numeričkom integracijom oskulatornih elemenata, pa se ne preslikavaju direktno u vrednosti koje odgovaraju čvorovima koordinatne mreže definisane u prostoru sopstvenih elemenata.*

**sopstveni elementi kretanja izvode se iz oskulatornih eliminacijom kratko- i dugo-periodičnih poremećaja.*

Rešenje (Milani 1993)

Fit modelom u kojem su frekvencije predstavljene kao polinomi u funkciji pogodnih kartezijanskih analoga sopstvenih elemenata, sa koeficijentima određenim metodom najmanjih kvadrata iz postojećeg kataloga sopstvenih elemenata i frekvencija asteroida. Kada se polinomi jednom odrede, unoseći u dobijene eksplicitne izraze koordinate čvorova mreže, frekvencije se direktno računaju i rezonance se zatim mogu grafički predstaviti konturnim linijama.

U praktičnoj primeni potrebni pripremni koraci:

- Izbor kataloga i dinamičkog modela
- Podela asteroidnog prstena na dinamički “uniformne” zone
- Odstranjivanje asteroida sa netačnim frekvencijama (N_{rdc})

Normalizovane kartezijanske promenljive imaju vrednosti u intervalu (0,1) bez obzira na izbor granica dinamičke zone:

$$x_i = 2\,\frac{a - x_0}{a_M - a_m}; \qquad x_0 = \frac{a_M + a_m}{2}; \qquad i = 1, .., N_{rdc}\,,$$

$$y_i = \frac{e_i}{e_M}; \qquad z_i = \frac{\sin I_i}{\sin I_M}\,; \qquad i = 1, .., N_{rdc}\,,$$

Table 3 Monomials of the polynomial fit. There are $N_{mon} = 4(6)$ monomials of degree 0 in variables (y, z), $N_{mon} = 12$ up to degree 2, $N_{mon} = 24$ up to degree 4, and $N_{mon} = 40$ up to degree 6. Two additional degree 0 monomials, x^4, x^5 apply to zones 4 and 9 only.

Deg in y, z	Monomials
0	$1, x, x^2, x^3, (x^4, x^5)$
2	$y^2, xy^2, x^2y^2, x^3y^2, z^2, xz^2, x^2z^2, x^3z^2$
4	$y^4, xy^4, x^2y^4, x^3y^4, y^2z^2, xy^2z^2, x^2y^2z^2, x^3y^2z^2, z^4, xz^4, x^2z^4, x^3z^4$
6	$y^6, xy^6, x^2y^6, x^3y^6, y^2z^4, xy^2z^4, x^2y^2z^4, x^3y^2z^4,$
6	$y^4z^2, xy^4z^2, x^2y^4z^2, x^3y^4z^2, z^6, xz^6, x^2z^6, x^3z^6$

$$\mathbf{m_g} = G \cdot (f' \cdot \mathbf{g}); \quad \mathbf{m_s} = G \cdot (f' \cdot \mathbf{s}),$$

where $\mathbf{m_g}, \mathbf{m_s}$ are vectors of solutions, G is the covariance matrix, f' is a transpose of the design matrix (without minus sign), and $\mathbf{g}, \mathbf{s}$ are vectors of asteroid secular frequencies as computed from the synthetic theory.

Koordinate na 2D mreži (treća koordinata fiksna):

$$a_g(i) = a_m + (i-1) \cdot a_{step}\,; \quad a_{step} = \frac{a_M - a_m}{N_a - 1}\,; \quad i = 1, N_a \tag{4}$$

$$\sin I_g(j) = \sin I_m + (j-1) \cdot \sin I_{step}\,; \quad \sin I_{step} = \frac{\sin I_M - \sin I_m}{N_I - 1}\,; \quad j = 1, N_I$$

Frekvencije na čvorovima mreže:

$$g_i(j) = \sum_{l=1}^{N_{mon}} p(j,l) \cdot m_g(l)\,; \quad s_i(j) = \sum_{l=1}^{N_{mon}} p(j,l) \cdot m_s(l)\,; \quad i = 1, N_a;\ j = 1, N_I\,.$$

Kontrola tačnosti

Odnos signal-šum pri određivanju koeficijenata polinoma:

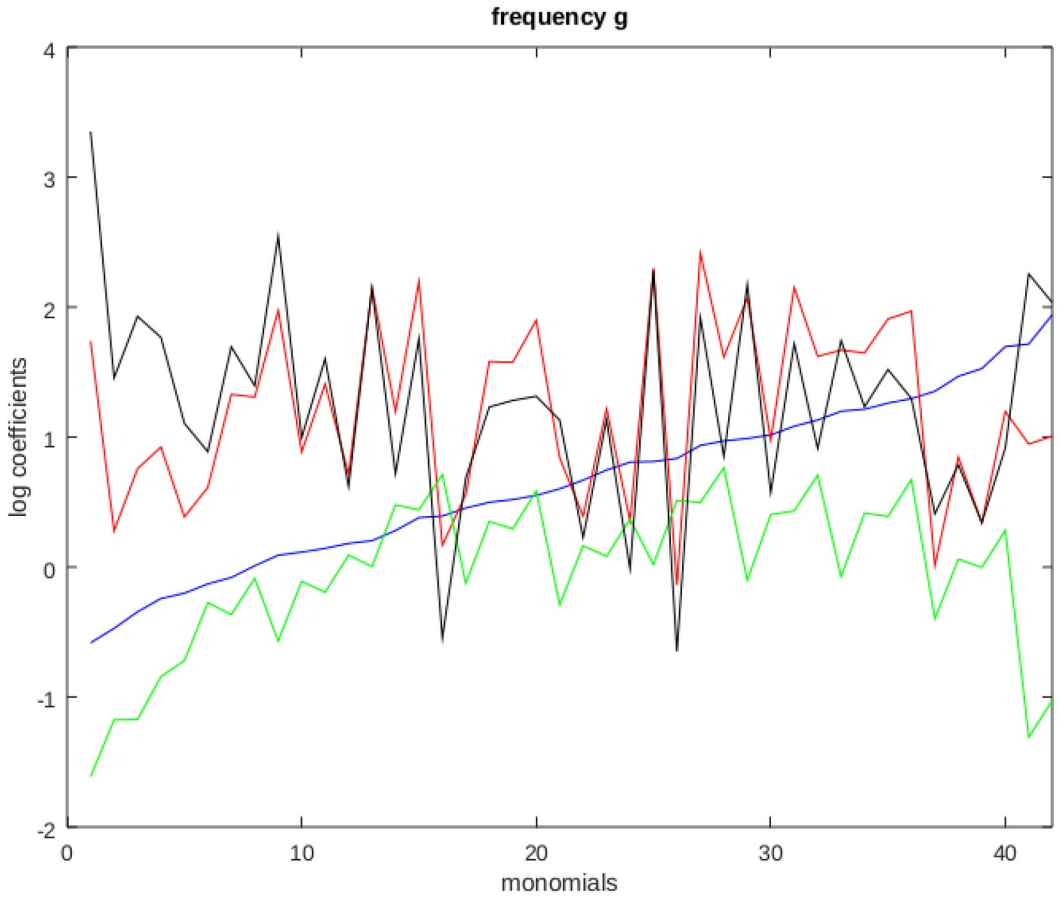
$$S_g(l) = \frac{m_g(l)}{\sigma(m_g(l))}\,; \quad S_s(l) = \frac{m_s(l)}{\sigma(m_s(l))}\,; \quad l = 1, N_{mon}.$$

Post-fit ostaci:

$$R(g)_i = g_i - \sum_{j=1}^{N_{mon}} f(i,j) \cdot m_g(j)\,,$$

$$R(s)_i = s_i - \sum_{j=1}^{N_{mon}} f(i,j) \cdot m_s(j)\,; \qquad i = 1, N_{rdc}$$

Zona 4 (2.502<a<2.701)



1/2 log sopstvene vrednosti normalizovane kovarijantne matrice (plava linija);

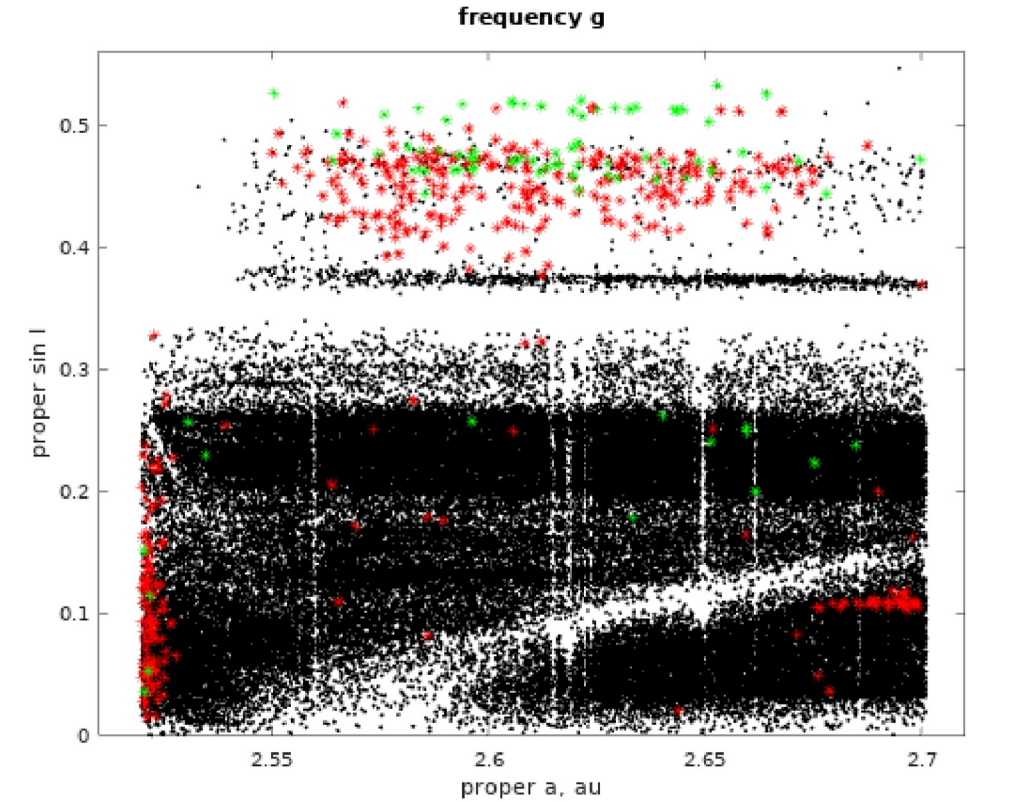
koeficijenti monoma (crvena linija);

standardna odstupanja koeficijenata (zelena linija);

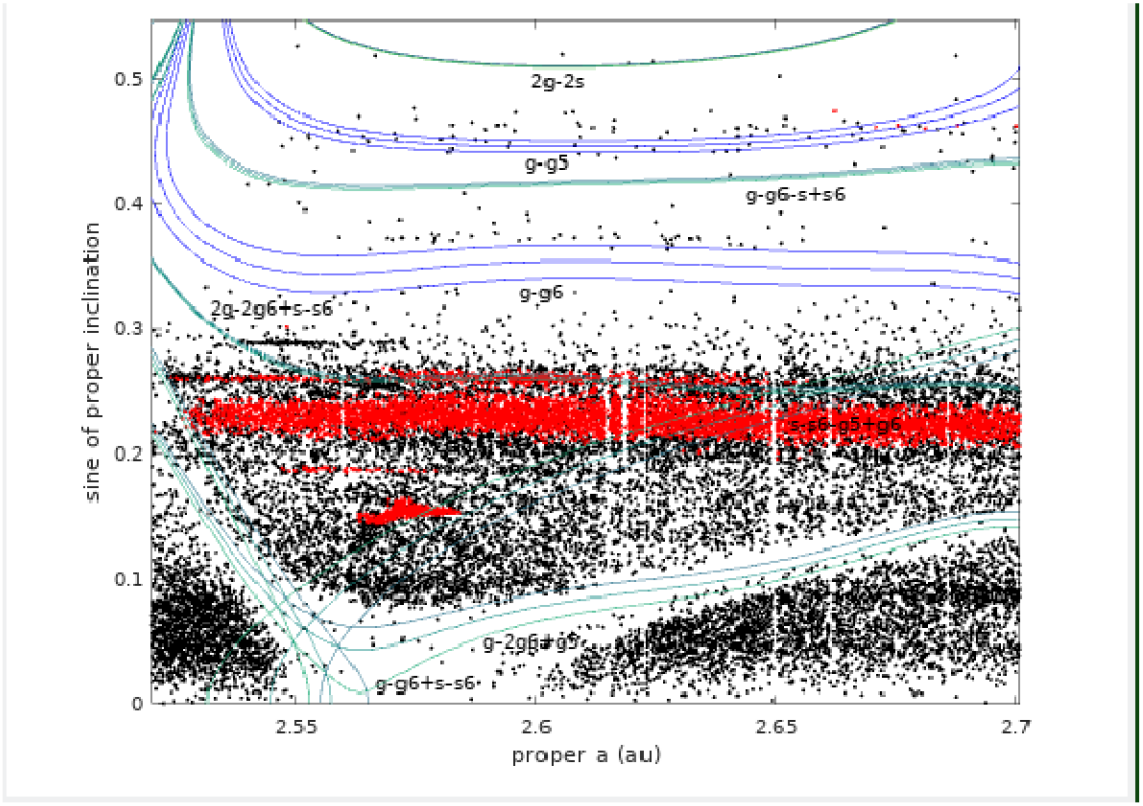
odnos signal-šum (crna linija).

Zona 4: valjanost fita

Freq.	σ_{all}	$N10$	σ_{10}	$N3$	σ_3	σ_{40}
g	1.1376	85	0.6795	542	0.5276	1.3152
s	0.1500	85	0.1049	882	0.0724	0.1552

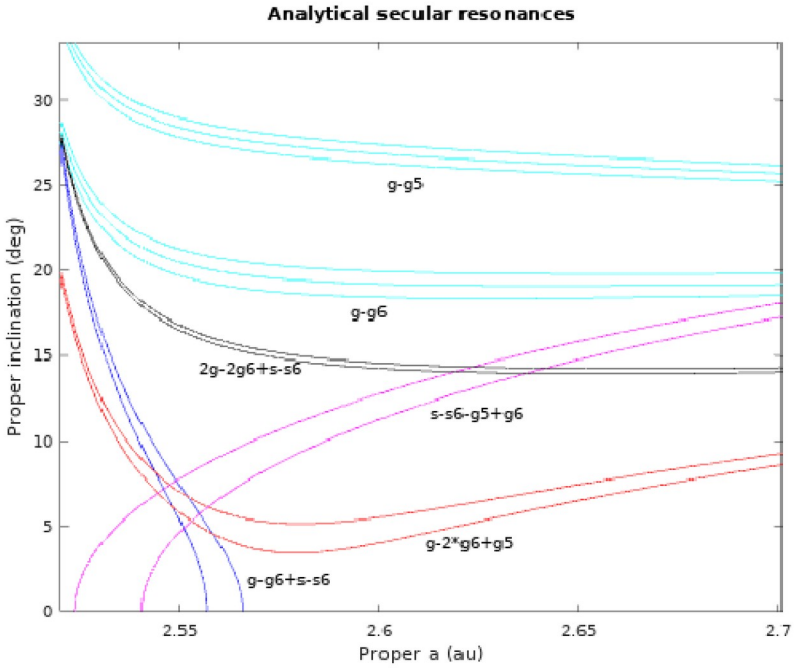


Zona 4: sekularne rezonance

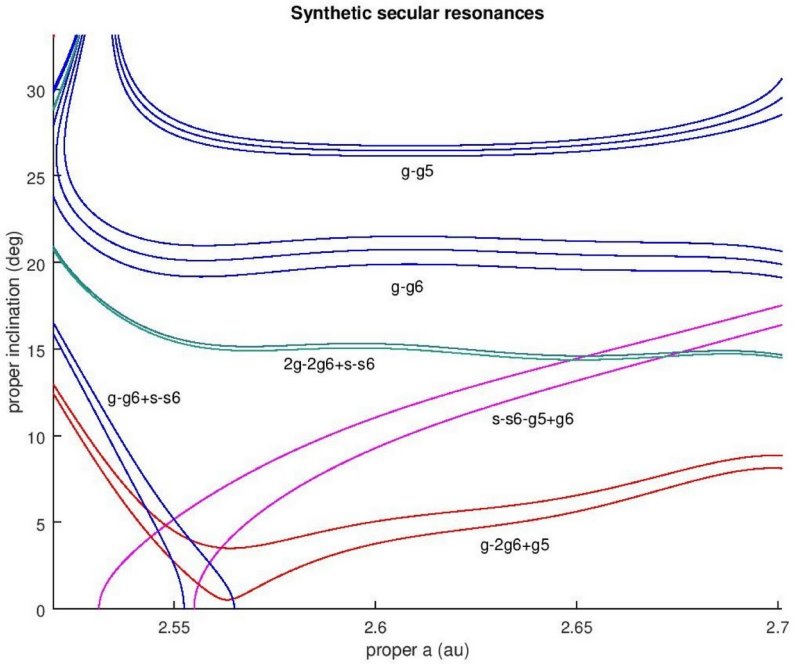


Background asteroids (black dots);

Family asteroids (red dots).

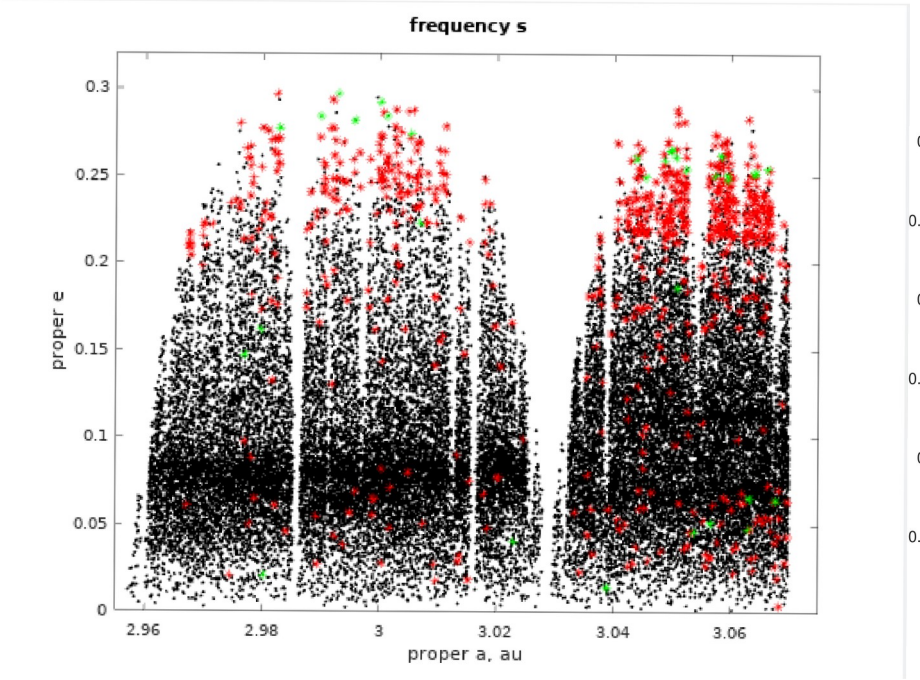


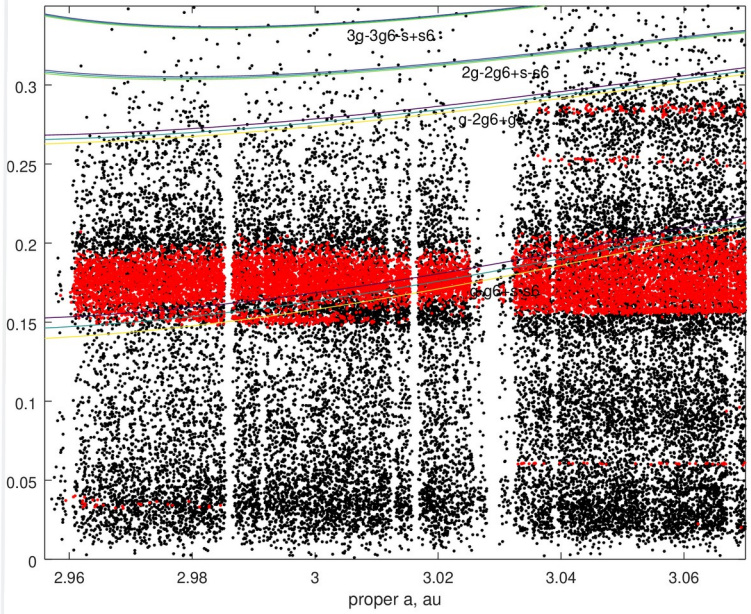
Analitičke vs. sintetičke sekularne rezonance



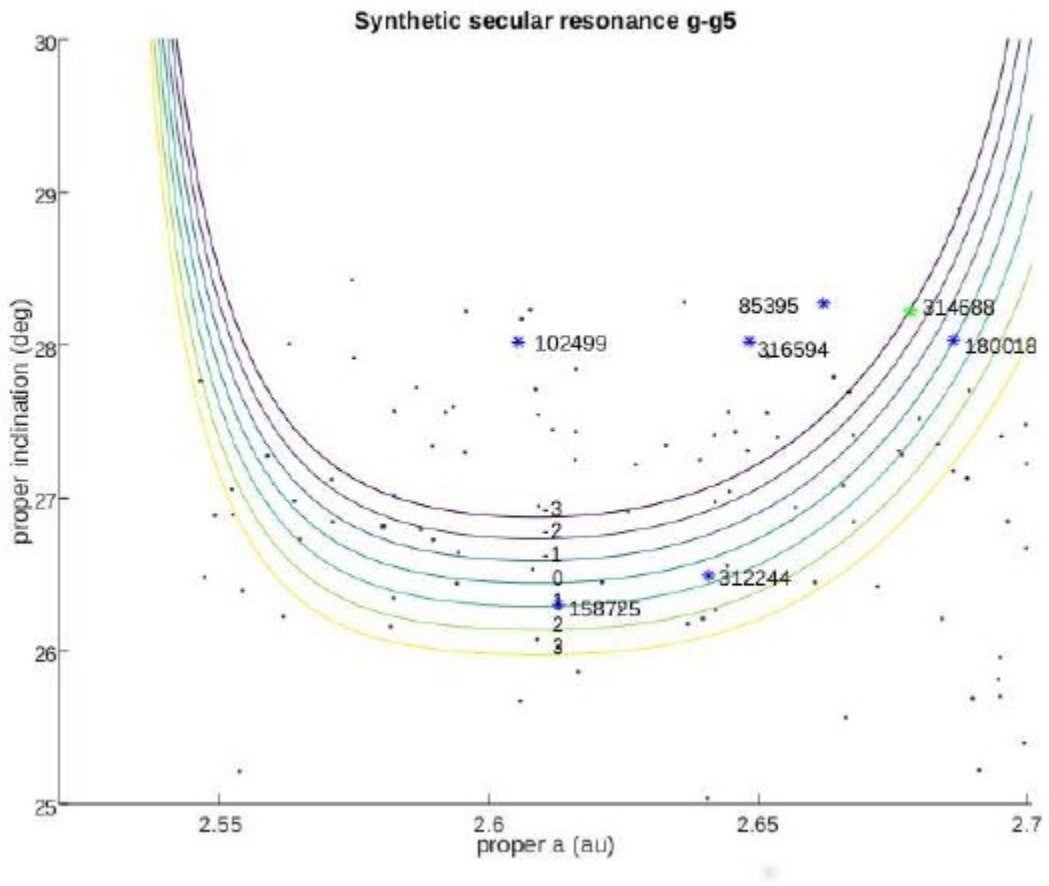
Zona 7 (2.956<a<3.070)

Freq.	σ_{all}	$N10$	σ_{10}	$N3$	σ_3
g	0.5627	19	0.2176	76	0.1997
s	0.1502	33	0.1399	784	0.0907





Sekularna rezonanca g-g_5 (e=0.13). Konturne linije [-3 -2 -1 0 1 2 3] “/y. Tačkice: svi asteroidi sa 0.105<e<0.155. Zvezdice: “rezonantni” objekti.



Asteroid	g
85395	5.1759
102499	4.4883
158725	5.5234
180018	5.6953
312244	4.0023
314688	3.8958
316594	5.2734

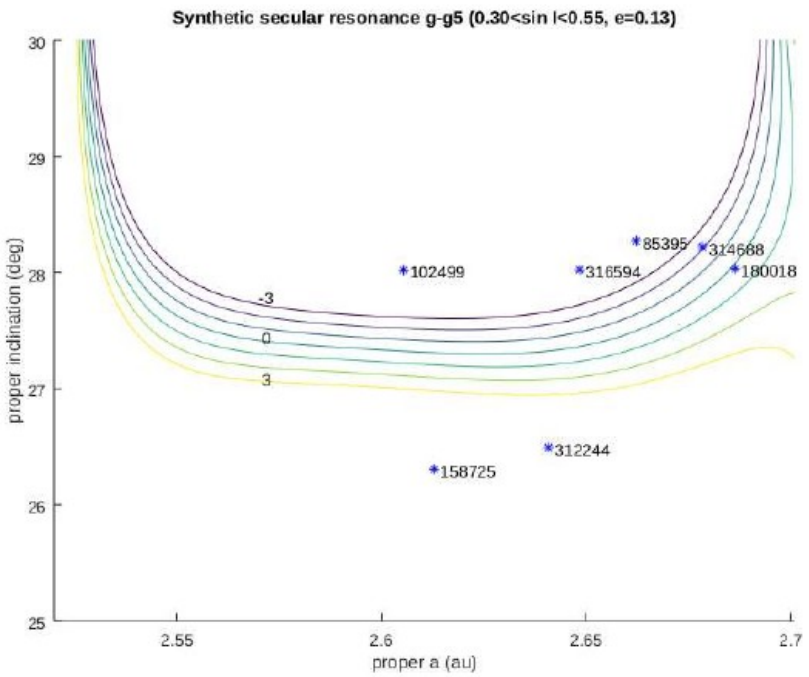
Netačan položaj rezonance

Pogrešni sopstveni elementi/frekvencije

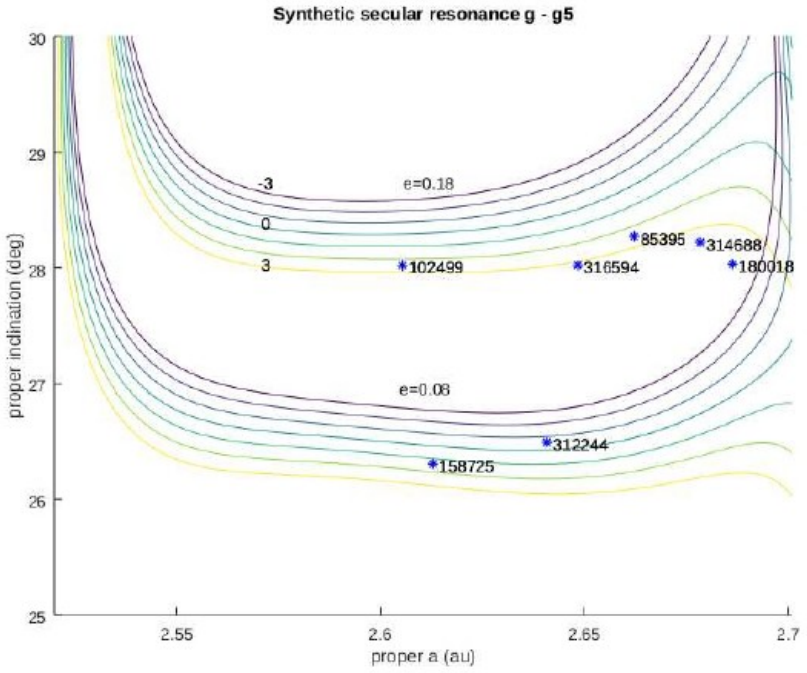
Mala snaga rezonance

Uticaj drugih rezonanci

Osetljivost položaja rezonance

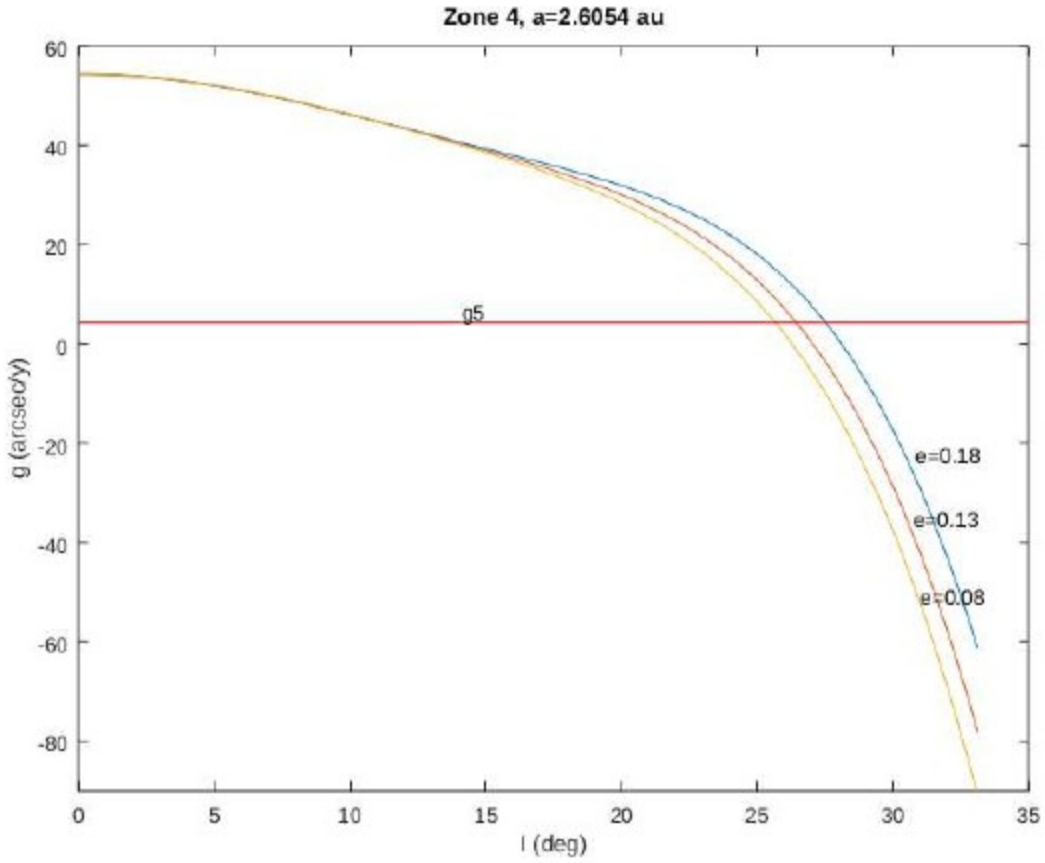


Samo asteroidi visokih nagiba
(0.30<sin I< 0.55)

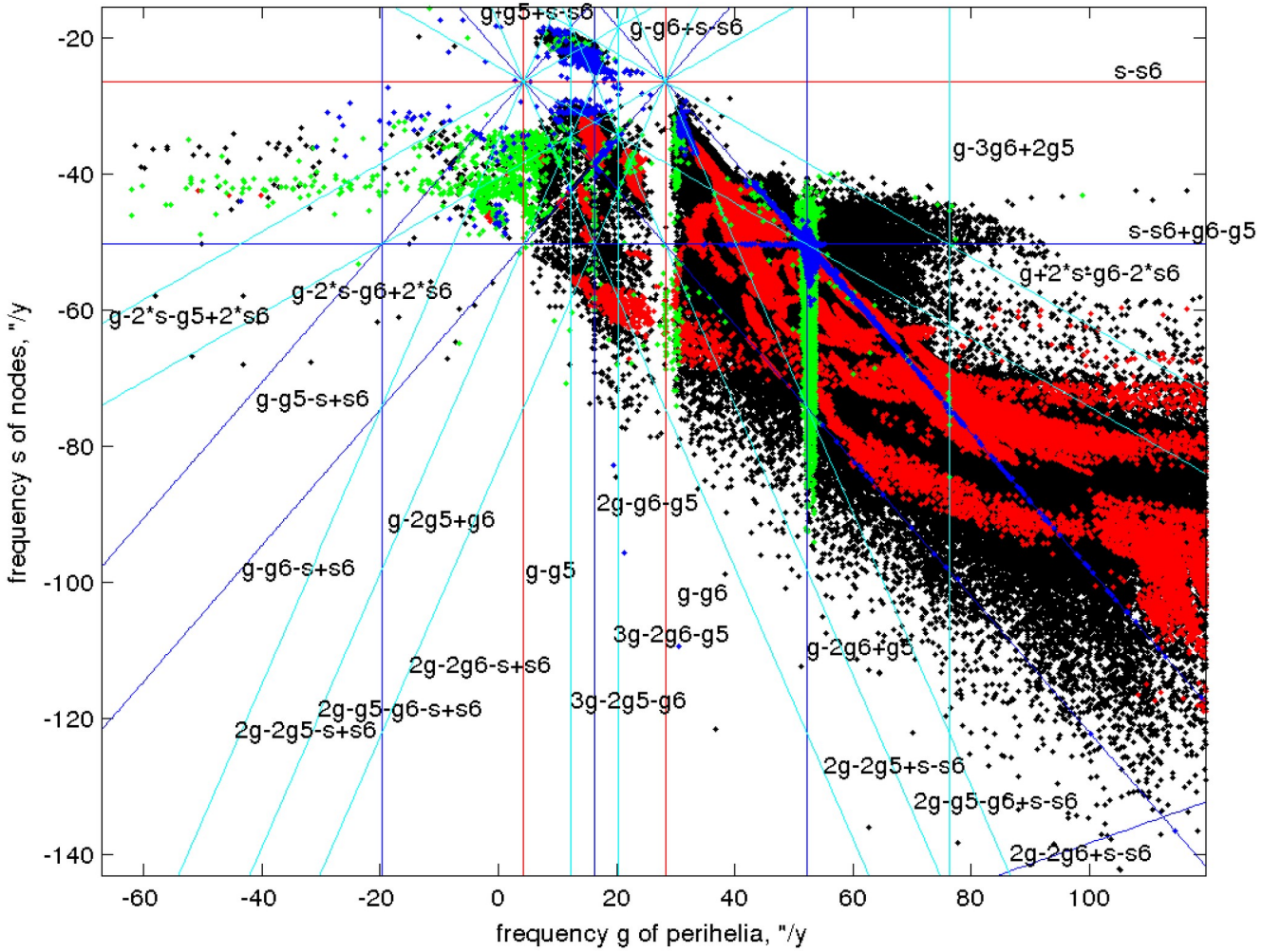


Dve vrednosti fiksirane koordinate
(e=0.08; e=0.18)

Profil promena frekvencije *g* u funkciji sopstvenog nagiba



Umesto zaključka: Glavni prsten i rezonance u prostoru frekvencija



Hvala!