

High-order mean-motion resonances in the main belt

Ivana Milić Žitnik

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- ▶ The structure of the main belt is largely affected by mean-motion resonances (MMRs):
 - They can serve as a source of dynamical chaos because of the overlapping of several resonances, but may also serve as long-term protection mechanisms in certain cases,
 - Within a mean motion resonance, the asteroid's semimajor axis undergoes periodic oscillations around the resonance center,
 - MMRs are capable of producing changes in asteroid orbital elements, especially in eccentricity and inclination,
 - An additional mechanism influencing the orbital evolution of asteroids is resonance sticking.
- ▶ This study aims:
 - to identify the fraction of the high-order two-body and three-body MMRs in the main belt compared to the known data in Smirnov & Shevchenko (2013) and Smirnov et al. (2017) the MMRs of the resonant order $q < 10$ for two-body MMRs and $q < 7$ for three-body MMRs,
 - to confirm statistically that at least temporarily resonant objects constitute a large fraction of the main belt,
 - to identify how many objects migrate from one resonance to another.

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- ▶ For the sample selection, we used the Minor Planet Center database (<https://minorplanetcenter.net/data>).
- ▶ For the numerical integration, we used the initial data from the NASA Horizons service.
- ▶ The numbered asteroids with $e < 0.35$, $2.0 \leq a \leq 3.5$ AU, observation arcs > 30 days.
- ▶ We selected randomly asteroids within each semimajor axis interval, with the number of selected objects depending on the local density.
- ▶ In total, we had 8672 orbits of asteroids to identify.
- ▶ The orbits of the asteroids were integrated over a span of 10^5 yr using the package resonances (Smirnov 2023), which utilizes rebound (Rein & Liu 2012) for the integration, considering all perturbations from all planets, with initial epoch 2024-03-31, with SABA(10,6,4) integrator.

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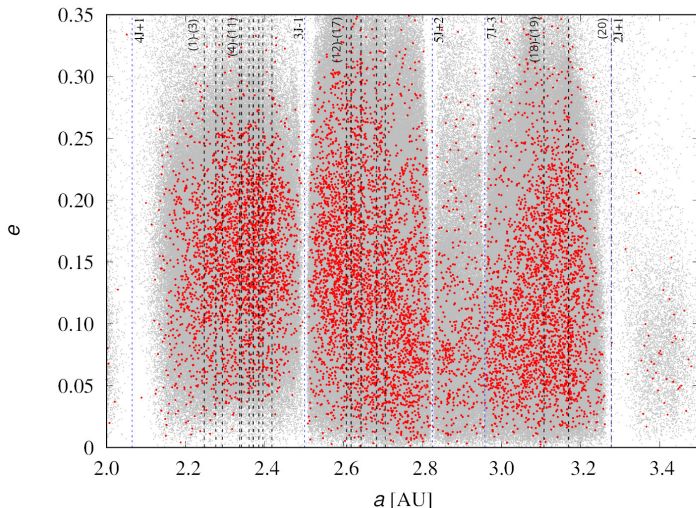


Figure: Main-belt asteroid distributions in (a, e) plane. Gray dots show objects from the Minor Planet Center database and red dots show the selected sample. Dotted black lines mark the twenty most populated two-body MMRs from Table 4. Blue vertical lines indicate five most important Kirkwood gaps in this region.

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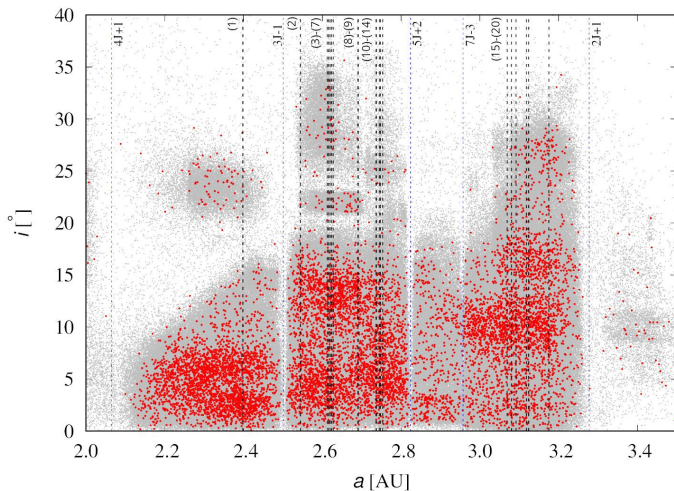


Figure: Main-belt asteroid distributions in (a, i) space. Gray dots show objects from the Minor Planet Center database and red dots show the selected sample. Dotted black lines mark the twenty most populated three-body MMRs from Table 2. Blue vertical lines indicate five most important Kirkwood gaps in this region.

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Notes. The “Initial” column corresponds to the output of the resonances package without any modifications. The “Final” and “% of Total” columns (Total = 8672) represent the final results after manual inspection (absolute and relative values, respectively). The data are provided separately for the pairs “asteroid–MMR” (named “pairs”) and unique asteroids. The variable s denotes the resonant status.

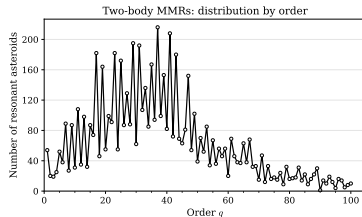


Figure: Distribution of resonant asteroids over resonance order q for two-body MMRs. The horizontal axis shows the resonance order q , and the vertical axis indicates the number of resonant asteroids. The drops for odd orders are explained by the smaller number of possible integer combinations in the resonant argument: they must be coprime, and for odd values only even integers are possible; otherwise, the greatest common divisor exceeds two.

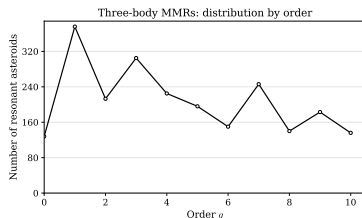


Figure: Same as previous figure, but for three-body mean-motion resonances.

Table 2. Top 20 three-body MMRs ranked by the number of resonant asteroids.

Resonance	Total	Pure	q
5J-2S-2	53	11	1
3J-2S-1	26	3	0
3J-1S-1	23	3	1
4J-3S-1	23	0	0
4J-2S-1	22	14	1
5J-7S-1	18	1	3
2M-10J-1	18	0	9
1J+4S-1	17	1	4
1J-9S+1	16	0	7
1M-9J+1	14	0	7
2M-7J-2	14	0	7
1M-2J-2	13	1	3
1M-5S-2	13	0	6
1M-9S-1	13	0	9
2J+2S-1	13	0	3
6J-1S-2	12	1	3
2M+2J-5	12	0	1
3M-5J-5	12	0	7
5J+7S-3	12	0	9
1J+3S-1	11	0	3

Notes. The “total” column lists the number of resonant asteroids, the “pure” column lists the number of pure librators, and the last column is the resonance order q .

Table 4. Top 20 two-body MMRs ranked by the number of resonant asteroids.

Resonance	Total	Pure	q
1M-2	39	2	1
27J-10	18	0	17
14J-5	17	0	9
8J+3	17	0	11
13J+4	16	0	17
1M-3	16	0	2
2J-1	15	11	1
8S-1	15	2	7
13J-6	15	0	7
13M-24	15	0	11
14J+5	15	0	19
24M-47	14	0	23
33S-4	14	0	29
7M-16	14	0	9
7S-1	14	0	6
14M+27	13	0	41
16J+5	13	0	21
21M-40	13	0	19
23M-42	13	0	19
35S+4	13	0	39

Notes. The “total” column lists the number of resonant asteroids, the “pure” column lists the number of pure librators, and the last column is the resonance order q .

Table 3. Same as Table 2 but sorted by the number of pure librators.

Resonance	Total	Pure	q
4J-2S-1	22	14	1
5J-2S-2	53	11	1
3J-2S-1	26	3	0
3J-1S-1	23	3	1
5J-6S-1	10	2	2
4M+3S-9	7	2	2
2M+7J-9	5	2	0
4M+3J-8	4	2	1
6J-8S-1	4	2	3
5J-7S-1	18	1	3

Table 5. Same as Table 4 but sorted by the number of pure librators.

Resonance	Total	Pure	q
2J-1	15	11	1
1M-2	39	2	1
8S-1	15	2	7
3M-7	8	2	4
6M-11	11	1	5
11J-5	9	1	6
27J-10	18	0	17
14J-5	17	0	9
8J+3	17	0	11
13J+4	16	0	17

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<https://doi.org/10.1051/0004-6361/202557400>.

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- ▶ The percentage of resonant asteroids is much higher than originally anticipated due to higher-order resonances. For the two-body MMRs, the fraction of resonant asteroids is 40.07% (even without Trojans) versus 2.0 – 5.0% obtained in earlier studies. The maximum in the distribution of the number of asteroids by resonant order is near $q = 36$.
- ▶ For the three-body MMRs, the number of asteroids trapped in high-order ($8 \leq q \leq 10$) is still significant and does not drop.
- ▶ A quarter of asteroids (25.57%) are trapped in several MMRs, either at the same time or through the phenomenon of resonance sticking, moving from one resonance to another, suggesting that this is a typical behavior.
- ▶ The current research has a few limitations.
 - We studied a sample of asteroids from the main belt.
 - The order of three-body MMRs was limited to 10.
 - We did not explicitly study other types of resonances (e.g., von Zeipel-Lidov-Kozai) within MMRs or the resonant multiplet.
- ▶ A systematic exploration of different types of resonances therefore represents an important direction for future research.

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Data availability

All data and code used in this study are publicly available. The resonances package source code is available at <https://github.com/smirik/resonances>. The validation datasets and example scripts are included in the package repository.

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THANK YOU FOR YOUR ATTENTION!